WORLD GOLD COUNCIL

Gold as a strategic asset for European investors

About the World Gold Council

The World Gold Council is the market development organisation for the gold industry. Working within the investment, jewellery and technology sectors, as well as engaging in government affairs, our purpose is to provide industry leadership, whilst stimulating and sustaining demand for gold.

We develop gold-backed solutions, services and markets, based on true market insight. As a result, we create structural shifts in demand for gold across key market sectors.

We provide insights into the international gold markets, helping people to better understand the wealth preservation qualities of gold and its role in meeting the social and environmental needs of society.

Based in the UK, with operations in India, the Far East, Europe and the US, the World Gold Council is an association whose members include the world's leading and most forward thinking gold mining companies.

For more information

Please contact Investment Research:

Marcus Grubb
Managing Director, Investment
marcus.grubb@gold.org
+44 20 7826 4724

Johan Palmberg
johan.palmberg@gold.org
+44 20 7826 4773

Juan Carlos Artigas
juancarlos.artigas@gold.org
+1 212 317 3826

Contents

About New Frontier Advisors

New Frontier is a Boston-based institutional research and SEC registered investment advisory firm specialising in the development and application of state-of-the-art investment technology. Founded in 1999, New Frontier principals, Richard Michaud and Robert Michaud, invented the world's first broad spectrum, provably effective, portfolio optimisation technology.

The patented Michaud Resampled Efficient Frontier™, described in *Efficient Asset Management: A Practical Guide to Stock Portfolio Optimization and Asset Allocation* (1st edition 1998 Harvard Business School Press, 2nd edition 2008 Oxford University Press), established a new world standard for effectively diversified and risk managed asset allocation and equity portfolio management. New Frontier holds three additional patents and has two patents pending in portfolio rebalancing, optimisation customisation, and derivative overlay management.

New Frontier's principals have over 60 years of institutional experience in consulting, asset management, financial planning, financial research, and investment technology development. New Frontier combines practical investment experience, patented techniques, and world class research skills to offer uniquely effective institutional quality consulting and investment advisory services as well as software licensing. New Frontier's investment funds are based on the rigorous application of advanced statistical and mathematical methods, investment theory, cutting edge technology and institutional quality asset management. The firm continues to develop unique applications of contemporary financial theory, mathematical statistics, and computer science. Through monographs, refereed academic and professional papers, patents, white papers, seminars, and invited presentations, New Frontier continues to pioneer new developments in portfolio management, investment strategy, and financial planning. New Frontier's services help institutional investors and advisors worldwide develop and manage more effective portfolios.

For more information contact:*

Richard Michaud
Robert Michaud
David Esch
Elise Schroeder

New Frontier Advisors, LLC
Boston, MA 02110, United States of America

T +1 617 482 1433
F +1 617 482 1434
E research@newfrontieradvisors.com

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Foreword from the World Gold Council

During a period of extraordinarily serious economic uncertainty in the euro area, we wanted to examine the relevance of gold as a strategic asset for euro-based investors to protect their portfolios and to mitigate the systemic risks being faced.

The euro area crisis has left investors around the world, both private and professional, anxious and perplexed as to the security of their assets and how to protect them from an increasing barrage of risks.

The World Gold Council commissioned New Frontier Advisors (NFA) to conduct this independent, proprietary research study using their respected optimiser, to explore the performance of gold within an efficient investment portfolio from the currency base and perspective of a euro-based investor.

The research from NFA makes an invaluable contribution towards the World Gold Council's own investment research, is particularly timely in view of the ongoing euro area crisis and supports the findings from recent reports on gold as a foundation asset, diversifier and hedge against risk, which can be found at www.gold.org.

The events of the last few years will undoubtedly have a lasting impact on global financial markets and regulation for years to come. We have witnessed the collapse of large financial institutions, the bailout of banks by national governments and unprecedented levels of monetary policy expansion. From late 2009, as national governments employed low interest rates and quantitative easing to stave off depression, already lofty levels of debt increased further. This precipitated two debt crises, the first since the foundation of the euro area.

In a world still littered with the debris of the banking meltdown, the euro area situation has further exacerbated a crisis of investor confidence. There has been a widening of bond yield spreads and the risk premia on credit default swaps between the indebted countries and other EU member states have risen to new highs. Stock markets around the world are experiencing elevated levels of volatility. The European Financial Stability Facility (EFSF) has eased the crisis but longer term solutions will involve structural change. Even if the crisis is averted, Europe faces the prospect of anaemic growth and protracted debt reduction as government spending is curtailed and consumers deleverage.

Many investors are struggling to regain their footing, not least in attempting to balance the need to generate returns in an environment of depressed real yields and tighter risk controls. Taking medium-term asset allocation decisions that may harvest upside potential, while ensuring that capital is protected on the downside, will be challenging, even for investors with deep resources.

Diversification theory underpins the mitigation of risk in portfolio management, and is particularly relevant when correlations increase. Both historically and statistically, gold has been consistently shown to mitigate risk and has again proven itself as a shield against the corrosive impact of an economic downturn and rising inflation.

While investors traditionally hold gold for its qualities as an inflation hedge and its secure status in times of trouble, what this research from NFA further endorses is its particular value as a source of diversification within a European investor's portfolio. When used as a foundation asset, gold preserves wealth. Gold helps to manage risk in a portfolio, not only by means of increasing risk-adjusted returns, but also by reducing expected losses incurred in extreme circumstances.

In the last decade we have lived through the effects of two of the worst bear markets in recent years and have seen the prospect of sovereign default in Europe – a previously unthinkable situation more reminiscent of Latin America during the 1980s. The relevance of gold as a strategic asset is likely to continue to grow as it has unique properties which can protect investors' portfolios from the corrosive effects of stagnation, negative real returns, high inflation and fiat currency debasement, in particular in the context of the euro area.

Executive summary

We examine the case for gold as a long-term or strategic asset allocation for European professional investors. The role of gold in asset management is currently very topical. From a longer term perspective a wide consensus exists that gold is a useful hedge against inflation and a long-term store of financial value.

Earlier research was hampered by methodological limitations and inappropriate assumptions. Michaud et al. (2006) was the first rigorous study to demonstrate the statistical significance of gold as a diversifying asset in an optimised asset allocation. The unique patented procedures allowed analysis of the statistical significance of gold within institutional mandates. However, the study was focused on dollar-centric investors. Prior to the early 1970s, the price of gold was fixed to the US dollar.

An effort to create a common currency among European nations begun in 1959 led to the establishment of the European Exchange Rate Mechanism (EERM) and the European Currency Unit (ECU) in 1979 and the euro in 1999. There now exists 25 years of useful index return history to evaluate gold in a strategic asset allocation.

We use long-term data and conservative assumptions with Michaud Resampled Efficient Frontier™ (MM) optimisation to address the significance of gold for European institutional strategic investors.

Our results demonstrate that gold may be a useful diversifying asset roughly comparable to small cap and emerging markets. An appropriate allocation depends on investor risk levels as well as the spectrum of candidate asset classes. A small though significant allocation of 1%-3% at low and moderate risk levels is recommendable under generally conservative assumptions in the context of a strategic asset allocation. Only weak evidence for an allocation to gold in high-risk strategic portfolios is available from our results. However, from a CAPM perspective, gold may be advisable as a diversifying factor of global economic risk in many well defined global strategic asset allocations.

For results summary and table: Gold allocation mean and 25% quartile estimates, see Table 6 page 14.

Introduction

Despite centuries of fascination and its current topicality, reasons for including gold in a portfolio strongly depend on investor habitat. Short-term tactical investors' interest in gold may be shaped by volatile or uncertain geopolitical events. Intermediate-term investors' attraction to gold may arise from higher demand for gold for jewellery and industrial applications in the context of inelastic supply. However, long-term strategic investors often do not invest in gold. Michaud et al. (2006) was the first published study to rigorously demonstrate that gold may have statistically significant diversification benefits in an institutional dollar centric strategic asset allocation context. This report examines the strategic case for gold from the perspective of European-based institutional strategic investors.

Gold is widely perceived to be a global default currency or store of monetary value that has inflation-hedging properties. Prior to the early 1970s and for much of the twentieth century, the price of gold was fixed to the US dollar.

In 1979, the effort to create a common European currency resulted in the EERM and the ECU, and continued in 1999 when the euro was first implemented. The ECU began trading as a unit of account in the US in 1986. There are currently 17 nations with the euro as a common currency.

There now exist 25 years of useful asset class index return history from a common currency European investor perspective. Earlier studies were limited by shorter-term return history, methodological limitations and inappropriate assumptions. We use longest-term historical return data available, an institutional context, conservative assumptions and modern statistical technology including Michaud or Resampled Efficient Frontier™ (MM) optimisation to address the importance of gold for European strategic asset allocation. Our unique patented procedures allow analysis of the statistical significance of gold for adding diversification value.[1] Our results show that gold can be a significant component of a strategic asset allocation for many European long-term investors.

The structure of this article is as follows: In Section I, we discuss some of the relevant background of the development of the euro, and the historical significance of gold as an investment. In Section II, we describe the study framework and detail the indices we chose, and describe the five cases in this study. In Section III, we describe the various quantitative and statistical measures we used for these cases. In Section IV, we present in detail the results for these cases. Section V contains a summary and conclusions. We also include an appendix, which contains some technical details about some of the quantitative methods of this study, and detailed tables of the mean and variance statistics of all of the assets used in the studies.

1 Invented by Richard Michaud and Robert Michaud. US patents 6,003,018, 6,928,418, 7,412,414, 7624,060: Israel 138018. Worldwide patents pending. New Frontier Advisors LLC is worldwide licensee.

I: Background

A brief history of the euro

After the demise of the US dollar based Bretton Woods System in 1971, most members of the European Economic Community (EEC) agreed to maintain stable exchange rates (the European "currency snake"). In March 1979, this system was replaced with the ECU as a common unit of account. After 1986 changes in national interest rates were used to keep the currencies within a narrow range. The Economic and Monetary Union replaced the ECU with the introduction of a single currency, the euro, on January 1, 1999. Unlike the ECU, the euro is a real currency, although not all European Union member states participate.

A brief history of gold

Gold has been used as a unit of exchange or money to a greater or lesser extent for much of the history of civilisation. Under the international gold standard that existed for much of the 19th century until World War I, currency was backed by gold. After World War II, the convertibility of the dollar into gold at a fixed rate of US$35 per troy ounce underpinned the stability of the new financial order set up at Bretton Woods in 1945. By the late 1960s, inflationary pressures on the dollar could no longer be sustained and the chapter of fixed exchange rate regimes, including a fixed gold price, ended in 1971. As Chart 1 shows, the new world of floating exchange rates, oil supply shocks, and silver corner[2] during the late 1970s and early 1980s, collapse of the internet bubble in 2000, the global meltdown in 2008 and current global recession and sovereign debt crises resulted in a substantial increase in price over much of the period.

The investment value of gold

The price of gold is associated with a number of investor behavioural issues. Gold is widely perceived to be a store of globally priced fundamental value. Because gold earns no interest, rises in interest rates may be inversely related to the price of gold and vice versa. In times of economic uncertainty, the demand for gold is likely to rise.

Chart 2 displays the relationship between gold and the euro area CPI Index from 1986 to 2010. While the performance of gold and the CPI were broadly related, there were periods when deviations were significant.[3] In particular the last six years have seen the price of gold rise well beyond any rise in interest rates or inflation. Gold's recent history may be most indicative of excessive speculation, unusual economic uncertainty, impact of financial innovations and concerns of the euro's long-term viability. However, Siegel (2002) demonstrates that very long-term evidence is consistent with the more traditional pattern of gold as an inflation hedge. Levin and Wright (2006) provide an empirical model relating short-run deviations to long-term real value.[4] The presumption of a long-term relationship of gold as a hedge against inflation is the basis for the conservative strategic return expectation in our study that long-term real return is equal to zero. This assumption is also consistent with our focus on gold for reducing risk and not on enhancing expected return.[5] This assumption is equivalent to gold price being roughly constant in real terms over the very long run.

Gold economic drivers

Gold is also a commodity and luxury good. The economic drivers that underlie supply and demand provide useful risk management insights that rationalise the lack of correlation between the return on gold and many other assets. For the last 30 years, roughly 75% of gold purchased has been in the form of jewellery. The drivers of jewellery demand are complex and vary widely from country to country. The pure commodity demand for gold derives from its medical and industrial uses. This industrial or technological demand is driven by similar economic factors to those that determine the use of other commodities, including the availability or desirability of substitute materials. The geographical diversity of gold demand is another aspect underpinning price movements that tend to be independent of those of the main capital markets. The long lead times that constrain the responsiveness of gold supplied through mine production, coupled with international agreements regulating sales by central banks, mean that the main source of supply elasticity is recycled gold.[6]

2 Silver corner refers to an episode during this period in which a pair of investors accumulated a large amount of silver futures contracts, subsequently taking delivery and pushing prices up.

3 The results need to be considered in the context of a far from perfect estimate of euro area CPI for the period and unusual recent uncertainty relative to the euro and global economies.

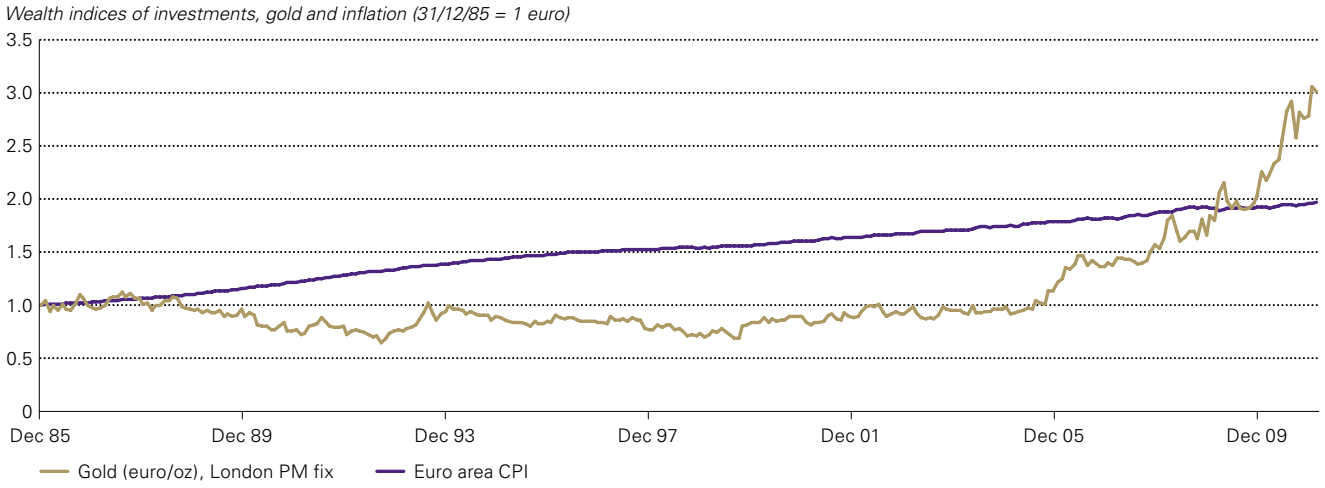
4 See also Burton et al. (2006) and Harmston (1998).

5 From classical financial principles such as the discounted cash flow model, since gold has no yield or other cash flow associated with ownership, its expected return is zero before storage costs.

6 For further discussion of the structure and dynamics of the gold market, see http://www.gold.org/investment/why_how_and_where/why_invest/demand_and_supply/.

Chart 1: Gold price 1969 to 2010

Chart 2: Gold versus euro area CPI 1985 to 2010



Strategic asset allocation

Asset management is generally defined in a core-satellite framework. Core or strategic investments are long-term focused whereas satellite or tactical strategies are shorter-term oriented.

Long-term investing is different from short-term because performance over time is associated with expected compound or geometric mean return. The expected geometric mean is positively related to expected return and negatively related to variance and time.[7] Added portfolio diversification increases long-term strategic asset allocation expected return.

The importance of strategic asset allocation is also explained in terms of the pioneering Brinson et al. (1986, 1991) studies. They examined the long-term performance of institutionally managed pension fund portfolios. They found that the single risk factor stock/bond mix explained more than 90% of the variance of performance. The two other explanatory components of performance – tactical asset allocation and fund selection – were of marginal importance and negatively related to long-term return. Their results are the basis for much of the consensus in the investment community that the average long-term risk policy of a fund is the single most important investment decision.

The importance of an asset in diversifying a portfolio is traditionally evaluated in the context of optimised portfolios on the Markowitz (1952, 1959) mean-variance (MV) efficient frontier. However, MV optimisation has a number of important limitations in investment applications. In particular, independent of investment value, any asset added to a MV optimisation appears to improve portfolio reward-to-risk. However, the allocations are not measures of statistical significance. The procedures used in this study are specifically designed to assess statistical significance and compensate for estimation error, to the extent that the data allow. They will be described in greater detail later.

Prior gold empirical studies

A number of studies have found that gold plays an important role in a MV optimised strategic asset allocation. However, many studies were based on largely unrealistic assumptions of the real long-term return premium for gold. For example Jaffe (1989) finds a significant role for gold in a MV optimised portfolio but assumes a real return of more than 12%.[8] More recently Idzorek (2005) reaches a similar conclusion but assumes a return premium of 6.3% relative to inflation for an equally-weighted index of gold, platinum, and silver. Both studies are based on traditional MV optimisation which has no facility for measuring the statistical significance of an asset in a MV optimised portfolio.

Michaud et al. (2006) was the first study to rigorously estimate the statistical significance of gold in a MV optimised portfolio. In contrast to prior studies, they assumed that gold has a zero real return. Their results showed that gold could be a statistically significant contributor to the diversification of an optimised institutional strategic asset allocation for low to moderate levels of portfolio risk. Our study will examine similar issues in the context of a European strategic asset allocation investor.

7 Michaud (1981, 2003).

8 Jaffe's data runs from September 1971 to June 1987. Over this time period gold was very nearly the best performing asset.

Commodity indices and gold

Strategic asset allocation is typically concerned with asset classes rather than individual assets. Gold stands apart as both an index and an individual asset and may not seem to fit into a traditional context for strategic asset allocation. The question of interest is what is to be gained from investing directly in gold over a well-diversified basket of commodities. One natural alternative is to consider gold in a basket of precious metals that includes silver and platinum. However, the size of the markets, liquidity issues, and relative homogeneity, argue that a basket of precious metals may have little additional institutional attraction relative to gold itself.

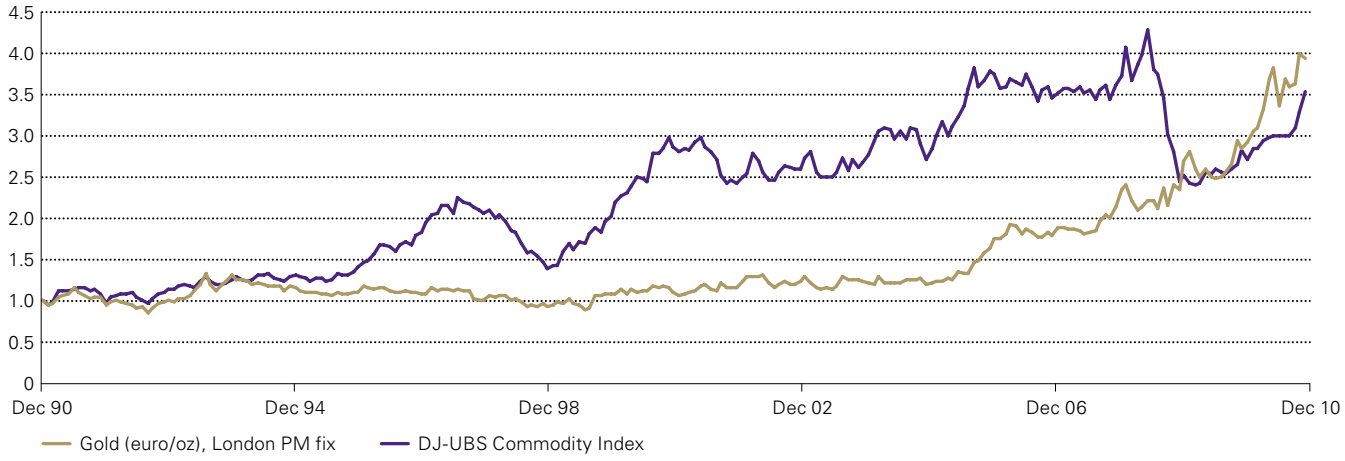
For reasons of commodity heterogeneity, including delivery, storage, and durability or its lack of, interest in diversified baskets of commodities is generally associated with commodity index futures. Three of the most popular indices for institutional investors are the S&P Commodity Index (S&P GSCI), the Dow Jones-UBS Commodity Index (DJ-UBS CI) and the Thomson Reuters/Jeffries CRB (Commodity Research Bureau) Index (TR/J CRB). Each index is weighted differently. For example the S&P GSCI is heavily weighted towards energy while the CRB is more equally weighted. The S&P GSCI has the broadest coverage, followed by the DJ-UBS CI and CRB. Each represents a very broad spectrum of commodity investment opportunities. For example, the S&P GSCI invests in 24 futures contracts; the DJ-UBS CI invests in 19 and the CRB in 19. In our study we examine the benefits of using the DJ-UBS Commodity Index Total Return for a European based investor.

Foresti and Toth (2005) reviewed the five components of commodity futures return. Gorton and Rowenhorst (2006) reported very attractive commodity index investment performance with low correlations with standard asset classes and risk-returns comparable to equity indices. However, advantageous weighting schemes and costless rebalancing were largely responsible for much of their observed performance. Nearly all the components of commodity future index returns are irrelevant for understanding gold price return. Commodity futures indices are often not very transparent while a gold price return index is a transparent and direct measure of the object of interest.

Chart 3 displays the performance history of gold relative to the DJ-UBS CI from 1990 to 2010. Empirical data strongly confirms the limitations of studying gold in the context of commodity index futures. While we shall include the DJ-UBS CI for comparison purposes, our focus is on the risk-return characteristics of the gold price return index relative to assets of interest for institutional strategic allocation. Whatever the investment merits of commodity indices, they are of little value as a useful surrogate for understanding the importance of gold in a strategic asset allocation. Although investor interest in commodities is growing and evolving rapidly, relatively few institutional investors have made major strategic allocation commitments to date. All of the studies detailed in Sections II to IV show negligible impact on the significance of gold when commodity indices are added to the portfolio. We will omit some presentations because the results are generally repetitious and of little additional interest.

Chart 3: Gold versus DJ-UBS Commodity Index 1990 to 2010

Wealth indices of investments, gold and DJ-UBS CI (31/12/90 = 1 euro)



II: Description of study framework, indices and cases

Study framework

Our study of gold as a strategic asset considers five sets of asset allocation studies. We include four studies using historical data spanning 1986 to 2010, and one study using the more recent 1999 to 2010 time frame. All returns are ECU or euro-based. The cases cover a progression from basic to an expanded set of asset classes. Long-term risk-return estimates are empirically based or conservatively estimated. Since many investors assume gold and commodities have inflation-hedging character, all risk-return estimates are euro area CPI-adjusted.

Some indices were not available throughout the entire term of a given study period. Our empirically estimated risk-returns use maximum likelihood estimation (MLE) to rigorously estimate the effects of missing data and avoid the many ad hoc practices often associated with incomplete return series estimates in a portfolio optimisation.[9] Our tests show that our MLE algorithm works well for estimating risk and return in the presence of missing data; we use it throughout the study. More details of which assets triggered MLE as well as supplementary assets used to assist in estimation are given in the descriptions of the cases later in this section.

All of the asset allocations presented here uses Michaud Resampled Efficient Frontier™ (MM) optimisation, a provably effective enhancement of MV optimisation.[10] For each data universe, three different sets of optimised asset allocations are performed: without gold and commodities, with gold and without commodities, and with both gold and commodities. In the cases with gold and commodities we present a sample of results. This is because the results provide little additional relevant information supporting the objectives of this study. It suffices to state that in all cases gold remains significant in the presence of commodities and its allocation is little affected. The Ledoit procedure for improved covariance estimation is used in all optimisation studies.[11] Non-negativity and budget constraints are imposed on all the optimisations. Michaud optimisation is described in more detail below, and some supplementary technical details about the various procedures are given in Appendix I.

Study indices

Indices were chosen for each asset class based on representativeness and availability. Total return index values are used for all asset classes except for MSCI international equities, which use net index values. All cases in this study are based in European currency units. Indices published in other currencies have been converted to a European base.

Our study begins in January 1986, a time period from which most major asset classes have available history. The appendix lists all asset classes, index names and start dates. The following paragraphs summarise our reasons for selecting these asset classes, and further information is contained in the detailed descriptions of the cases.

The J.P. Morgan (JPM) Government Bond Indices are good representations of the European bond market and were chosen over alternative indices due primarily for their historical availability. The Barclays Capital Aggregate Bond Index is a standard choice to represent the US fixed income market. JPM European Corporates, though only beginning in 1999, provide useful representation of the European corporate bond market and Barclays Capital Euro Overall All Maturities TR Index is the longest lived representation of European inflation-linked bonds.

MSCI indices are standard representations of both large and small capitalisation international equities. The MSCI European Small Cap Index is available as net data from December 2000, and is available only as price return index data from January 1993. Likewise, MSCI Emerging Index is available as net data from December 2000, with data from January 1988 being gross. Russell 1000 and 2000 indices are preferred representations of the United States large and small capitalisation markets for many purposes. The FTSE EPRA/NAREIT Developed Europe Index and the DJ US Select REIT Index are both standard representations of the European and US real estate markets respectively.

The breakdown of asset classes into subgroups in the expanded case resulted in use of asset classes with less historical data. To better estimate returns and risks of those asset classes in our expanded case with shorter histories, we include similar assets with longer existing histories in our MLE procedure. JPM offers German bonds broken down into maturity groups of 1-3, 5-7 and 7-10 years and European bonds broken down into wide maturity groups of 1-5 and 1-10 years. These all extend back through 1986 and are added as a supplement to the shorter lived JPM GBI European 5-7 and 10+ year bonds. Bank of America Merrill Lynch US Corporate bonds are added as supplemental data for the shorter lived European corporate index.[12]

9 See Appendix I for further description.

10 Michaud optimisation is not to be confused with the Feldman (2003) procedure used in Idzorek (2005). Michaud optimisation is patented technology and can be rationalised with rigorous proofs of likely superior out-of-sample performance. See Michaud and Michaud (2008a,b) for more information.

11 The sample covariance matrix estimate is known be highly error prone. Ledoit uses a shrinkage procedure to properly manage extreme high and low covariance elements toward a more sensible central belief. See Ledoit and Wolf (2003, 2004) and Michaud and Michaud (2008a) for further discussion.

12 DJ US Select REIT, DJ-UBS CI, and BarCap Aggregate Bond Index from Morningstar Principia (2011); MSCI indices from MSCI.com (2011); Russell indices from Russell.com and Morningstar Principia (2011). All other data provided by the World Gold Council via Bloomberg and JPM data subscriptions.

Case 1: Base case asset classes and empirical risk-return estimates

We begin our study of the diversification properties of gold within a European institutional strategic asset allocation context with respect to a minimal set of eight standard asset classes. These include: JPM Euro 3-Month T-bills, JPM GBI European bonds, BarCap US Aggregate bonds, MSCI Europe large and small capitalisation equities, MSCI World ex Europe, DJ-UBS Commodities and London PM fix gold indices. The European CPI inflation-adjusted average returns, standard deviations, and correlations are given in Table 1. In this base case, the MLE algorithm is necessary due to the shorter histories of the T-bill, European bond, European small cap and commodities indices.

Table 1: Empirical risk-returns estimates: basic assets classes, inflation adjusted; January 1986 to December 2010[1]

Asset name	Return[2]	Standard deviation[2]	Correlations							
			JPM Euro 3-Month T-bill	JPM GBI European Bond	BarCap US Agg Bond	MSCI Europe	MSCI Europe Small	MSCI World ex Europe	Commodities, DJ-UBS CI	Gold, London PM fix
JPM Euro 3-Month T-bill	2.6%	1.1%	1.00							
JPM GBI European Bond	4.4%	4.6%	0.45	1.00						
BarCap US Agg Bond	3.6%	10.8%	0.21	0.35	1.00					
MSCI Europe	6.7%	16.9%	0.02	0.15	0.22	1.00				
MSCI Europe Small	7.8%	18.6%	-0.03	0.06	0.16	0.87	1.00			
MSCI World ex Europe	5.0%	17.5%	0.03	0.16	0.45	0.80	0.70	1.00		
Commodities, DJ-UBS CI	3.4%	15.3%	-0.09	-0.02	0.36	0.32	0.35	0.45	1.00	
Gold, London PM fix	2.9%	15.7%	0.00	0.05	0.39	0.00	0.03	0.18	0.41	1.00

1 JPM Euro T-bill from November 1986, JPM GBI European Bond from February 1986, MSCI Europe Small from January 1993, DJ-UBS CI from January 1991.
2 MLE and Ledoit estimated, annualised from monthly return data.

Case 2: Strategic return premiums

Historical risk-return estimates are period dependent and often include assets that reflect ephemeral effects and unlikely future values. From the perspective of a long-term strategic asset allocation, period dependency should be minimised. Our objective is to use empirical data most advantageously for estimating the underlying long-term structure of global capital markets. While truly long-term data may be desirable the 1986 to 2010 historical period in Table 1 is currently the best possible given our objectives.

A useful solution is to use conservative return premium assumptions consistent with available long-term data and the presumed role of gold as an inflation-hedge. The more conservative the assumptions the more likely any significant findings may be reliable for long-term investing.

Table 2 summarises our strategic inflation-adjusted return premium assumptions for the assets in the base case studies. T-bill inflation-adjusted return for much of the 20th century is less than 1%.[13] As a conservative estimate, a zero real return for three month T-bills seems reasonable. If this assumption is in error it is unlikely to affect our results in any significant way. Following Siegel (2002), we assign a zero real return to gold. Since gold is the primary focus of our study, a zero real return will free our results from any special pleading bias critique. Note that the standard error of the mean for gold of 3.5% implies that zero real return is statistically plausible for the Table 1 data. A zero real return for an equal-weighted basket of commodities seems reasonable from a strategic perspective and consistent with our focus on gold in the context of more traditional strategic asset classes. For the remaining assets we use traditional return premiums for bonds and equities roughly consistent with long-term empirical return studies.[14]

Note that, as in Table 1, empirically small cap stocks have exhibited a positive return premium relative to large cap. But the actual observed premium can be very dependent on the small cap index used as well as time-period studied.[15] Given our predisposition for conservatism and the fact that gold is the major focus of our study, such an assumption should have little impact on our results in this case.

Table 2: Basic assets, strategic return premiums, inflation adjusted risk-return estimates; January 1986 to December 2010[1]

Asset name	Return	Std. dev.[2]
JPM Euro 3-Month T-bill	0.0%	1.1%
JPM GBI European Bond	1.5%	4.6%
BarCap US Aggregate Bond	1.5%	10.8%
MSCI Europe	6.0%	16.9%
MSCI Europe Small	6.0%	18.6%
MSCI World ex Europe	6.0%	17.5%
Commodities, DJ-UBS CI	0.0%	15.3%
Gold, London PM fix	0.0%	15.7%

1 JPM Euro T-bill from November 1986, JPM GBI European Bond from February 1986, MSCI Europe Small from January 1993, DJ-UBS CI from January 1991.
2 Ledoit estimated, annualised from monthly data.

13 Dimson et al. (2011).

14 Dimson et al. (2011), and Ibbotson (2010).

15 For example the Ibbotson Associates US small cap index over the 1974 to 2005 period has roughly a 5% premium over large cap, whereas the Russell US index premiums are closer to 2% or less. Also the results are often very time-period dependent.

Case 3: Expanded asset classes

Institutional strategic investors' actual portfolios generally include a number of additional asset classes beyond those in our base case. In our expanded cases we separate important subclasses of the asset classes in the base case and add a number of additional ones. European bonds are split into intermediate- and long-term bonds and MSCI World ex Europe is broken out into US large and small capitalisation equities, emerging markets and Pacific markets. Additionally, European corporates, European inflation-linked bonds, European and US real estate are added to the case. This more comprehensive set of assets is a reasonable approximation to contemporary standards for institutional strategic asset allocation practice.

Consistent with the base case, the expanded asset study spans January 1986 to December 2010. The expanded group of assets, however, have substantially more missing data over this time period. To better estimate returns and risks of those asset classes in our expanded case with shorter histories, we include similar assets with longer existing histories in our maximum-likelihood risk-return estimation process. We find that adding these ancillary data sources provides useful information for better estimating historical risks and returns for those assets with much missing data. The additional assets are added solely for supporting the estimation process, and are excluded from subsequent steps in the allocation process. The additional assets used only for estimation are: JPM European bonds, JPM GBI European 1-5 year bonds, JPM GBI European 1-10 year bonds, BofA Merrill Lynch US Corporate Bonds, MSCI World ex Europe, JPM Germany 1-3 year bonds, JPM Germany 5-7 year bonds and JPM Germany 7-10 year bonds. The complete set of asset classes with historical risks, returns, and correlations is found in Table A1 in Appendix I. For informational purposes the pairwise correlations for these assets is shown in Table A2, also in the appendix. The risk and return estimates of the asset classes in the expanded class optimisation study are taken from the estimates in Table A1 and shown in Table 3.

Table 3: Expanded assets, inflation-adjusted risk-return estimates; January 1986 to December 2010

Asset Name	Return*	Std. dev.*
JPM Euro 3-Month T-bill	2.6%	1.1%
JPM GBI European 5-7 Year	4.5%	4.4%
JPM GBI European 10+ Year	5.6%	8.0%
BarCap US Aggregate Bond	3.6%	10.8%
JPM Euro Corporates	4.2%	3.8%
BarCap Euro Inflation Linkers	2.9%	7.3%
MSCI Europe	6.7%	16.9%
MSCI Europe Small	6.7%	18.7%
Russell 1000 US Large	7.2%	18.9%
Russell 2000 US Small	7.2%	22.5%
MSCI Emerging	13.8%	26.6%
MSCI Pacific	3.9%	21.1%
European Real Estate	3.4%	16.1%
US REITs	6.9%	20.9%
Commodities, DJ-UBS CI	4.2%	15.5%
Gold, London PM fix	2.9%	15.7%

* MLE and Ledoit estimated, annualised from monthly data.

Case 4: Expanded asset classes with strategic return premiums

Table 4 provides a summary of the inflation-adjusted strategic return premiums associated with the expanded asset class studies. Adding more assets in a strategic asset allocation implies more estimation error in the optimisation process. Our bias is to assign conservative strategic premiums to the additional assets. REITs are a hybrid security having characteristics of fixed income instruments and equities. For simplicity we assume a strategic return halfway between large cap and long-term government bonds. The risk and correlation estimates are exactly as in Table A1 in Appendix I.

Table 4: Expanded assets, strategic return premiums, inflation adjusted risk-return estimates; January 1986 to December 2010

Asset name	Return	Std. dev.*
JPM Euro 3-Month T-bill	0.0%	1.1%
JPM GBI European 5-7 Year	1.5%	4.4%
JPM GBI European 10+ Year	2.5%	8.0%
BarCap US Agg Bond	1.5%	10.8%
JPM Euro Corporates	1.5%	3.8%
BarCap Euro Inflation Linkers	1.5%	7.3%
MSCI Europe	6.0%	16.9%
MSCI Europe Small	6.0%	18.7%
Russell 1000 US Large	6.0%	18.9%
Russell 2000 US Small	6.0%	22.5%
MSCI Emerging	6.0%	26.6%
MSCI Pacific	6.0%	21.1%
European Real Estate	4.3%	16.1%
US REITs	4.3%	20.9%
Commodities, DJ-UBS CI	0.0%	15.5%
Gold, London PM fix	0.0%	15.7%

* Ledoit estimated, annualised from monthly data.

Case 5: Expanded asset classes from 1999

The period from January 1, 1999 to December 31, 2010 is of special interest since it marked the formal use of the euro as a unit of currency in the euro area. Additionally, the period has a complete set of historical return data for our expanded set of asset classes. However, the period is anomalous in that gold was the highest performing asset with an average real return of 11.4%. While gold was an outstanding investment during this time period, using its historical return in our optimisations would be inconsistent with objectives, which is to examine its value as purely a diversifying asset in a strategic portfolio context. Table 5 provides a summary of the historical risk and return estimates for the past 12 years where the real return to gold is set to zero. A zero real return avoids the period specific character of a likely unreliable future return.

Table 5: Expanded assets, inflation adjusted risk-return estimates; January 1999 to December 2010

Asset name	Return	Std. dev.[1]
JPM Euro 3-Month T-bill	1.2%	1.2%
JPM GBI European 5-7 Year	2.8%	3.9%
JPM GBI European 10+ Year	2.9%	7.0%
BarCap US Agg Bond	2.9%	10.4%
JPM Euro Corporates	2.6%	3.6%
BarCap Euro Inflation Linkers	3.1%	4.9%
MSCI Europe	1.4%	16.6%
MSCI Europe Small	7.5%	19.8%
Russell 1000 US Large	0.7%	17.3%
Russell 2000 US Small	5.7%	21.7%
MSCI Emerging	13.1%	23.3%
MSCI Pacific	2.9%	17.5%
European Real Estate	6.9%	17.7%
US REITs	10.0%	23.6%
Gold, London PM fix	0.0%[2]	16.4%

1 Ledoit estimated, annualised from monthly data.
2 Real return set to zero for consistency with report objectives.

III: Quantitative procedures

Michaud optimisation

Markowitz MV optimisation is the most prevalent institutional framework to define strategic asset allocations. MV optimised portfolios, however, are well known to be unstable and have poor out-of-sample performance characteristics. Even small changes in the inputs often lead to large changes in the optimised portfolios. MV optimisation is unstable because it implicitly assumes that all risk-return inputs are perfectly known, ie. estimated without error. Even small amounts of estimation error can perturb the results greatly. Investment information is endemically very uncertain and highly inconsistent with the accuracy implied by feeding one set of inputs into a MV optimisation. As a result MV optimised portfolios are unreliable, and unsuitable as a framework for understanding investment value including the role of gold.

All portfolio allocations in this study are computed using Michaud (MM) optimisation (Michaud 1998, Michaud and Michaud, 2008a, 1998). MM optimisation is an extension of Markowitz Mean-Variance (MV) optimisation which improves out-of-sample performance (Michaud 1998, Ch. 6, Markowitz and Usmen 2004, Michaud and Michaud 2008, Chs. 6 and 9). It starts with the same mean and variance assumptions as MV, but treats them as uncertain estimates of unknown parameters rather than as given truths. It then simulates many plausible values for the true means and variances and averages together their associated efficient frontier portfolios to produce the result. The MM frontier is stable and consists of optimally diversified portfolios representing a range of investor risk preferences. An important feature of MM optimisation is that the statistical significance of an individual asset within any frontier portfolio can be examined using the individual MV simulations.

Several recent advances have occurred in improving the expected out-of-sample performance of MM optimisation, and these methods are used to full advantage in the optimisations shown here. Sections A1 to A4 in the appendix describe (1) the missing data procedure, (2) the *t* distribution for resampling, (3) the Critical Line Algorithm, and (4) Arc Length Association Method.

Statistical significance estimation

In the exhibits that follow we display the estimated statistical significance of gold in strategic asset allocations relative to the portfolios on Michaud efficient frontiers. Michaud et al. (2006) presented the 10th and 90th percentiles of the simulated gold allocations for a US investor. These bounds can then be interpreted as an 80% confidence interval for the allocation to gold. More particularly, they focused on the 10% lower bound to determine whether the allocation to gold is significantly different from a zero value. A zero 10% quantile is evidence that the allocation may not be significant at the 10% level and conversely, a nonzero bound indicates a statistically significant allocation. The 2006 dollar-centric study, based on long-term well established data series, was able to conclude that gold was significant at the 10% level with at least 1%-2% allocations for low to moderate risk levels on the efficient frontier.

Our current asset class data extends from January 1986 to December 2010 and represents roughly all the useful available returns for estimating the investment value of gold for institutional quality European strategic asset allocation strategies within a common currency context. While risk-return estimates based on 25 years of data contain much valuable information, they can hardly be considered definitive as a description of long-term capital market structure or as reliable forecasts of future performance of gold in a euro-based strategic asset allocation.

Besides longevity, our data also has additional limitations for addressing this study's objectives. Both the ECU and the euro have required significant interventions over the course of their 25-year history. Even recent history of the euro includes significant accommodations as well as continuing uncertainty of its long-term viability in its current state. Index return data for the combined nation-state economies of the euro area may represent less reliable statistical information than the US. In addition, our estimates of real rates of returns are based on the euro area CPI index. Inflation is difficult to estimate for any unified country and far more for the combined euro area.

Because of the limitations of our data, we use a 25% lower bound to determine whether the allocation to gold is statistically significant. This is a much lower standard for significance than the more traditional 5% or 10% levels generally associated with large scale statistical studies or the 10% level used in the US dollar-centric Michaud et al. (2006) study. However, a failure to establish 10% significance may neglect the considerable evidence in favour of gold as an important portfolio constituent for diversifying risk. Indeed, many of the other portfolio assets in the study would also fail to show significance at 10% for this data.

We employ very conservative estimates of return and rigorous estimates of risk to enhance the reliability of our results. The results we observe in our current study do not contradict those we found from the more extended data of the Michaud et al. (2006) study. We display 25% results over a wide range of scenarios and consider the evidence indicative given its consistency over a wide array of different investment assumptions and universes. Nevertheless, it remains an important consideration that no higher level of standard of significance for gold exists in our analysis.

IV: Results

Summary

Table 6 provides a simple summary of the results of the study. The table displays the mean and lower quartile allocations for gold for three different investor risk levels along the efficient frontier. The lower quartile results are given in parentheses next to the mean allocation. Because of the statistical limitations of the data, strong levels of significance are not available. We use a nonzero lower quartile allocation as indicative of statistical evidence for the importance of gold as a diversifying asset in a strategic portfolio.

In all cases the results indicate nonzero mean allocations and generally nonzero lower quartile values except at some high-risk levels. The results are most favourable for gold in the two relatively institutionally limited basic assets cases. In the three more institutionally relevant expanded assets cases the role of gold is diminished but not insignificant under our assumptions. In all cases our results should be interpreted as rough guidelines and not exact recommendations.

Table 6: Results summary

Gold allocation mean (25% quartile estimate)				
		Investor risk profile equity / fixed income ratio		
Asset universe	Time span	Low 25% / 75%	Medium 50% / 50%	High 75% / 25%
Basic assets (Case 1)	1986 to 2010	5.4%[2] (1.3%)	9.3%[2] (1.9%)	10.1%[2] (1.3%)
Basic assets, strategic return premiums (Case 2)	1986 to 2010[1]	3.5%[2] (0.6%)	5.5%[2] (0.8%)	6.4%[2] (0.7%)
Expanded assets (Case 3)	1986 to 2010	2.4%[2] (0.3%)	3.3%[2] (0.1%)	3.0% (0.0%)
Expanded assets, strategic return premiums (Case 4)	1986 to 2010[1]	2.6%[2] (0.4%)	3.3%[2] (0.2%)	3.1% (0.0%)
Expanded assets (Case 5)	1999 to 2010[1]	2.2%[2] (0.2%)	3.7%[2] (0.2%)	3.78%[2] (0.1%)

1 In these cases gold has been assigned a zero real return.
2 Significant at the designated 25% level of this study.

Case 1 results

We examine three cases associated with our base case set of data in Table 1. The first represents a baseline that excludes gold and commodities on MM optimised asset allocations. The second case includes gold and the third includes both gold and commodities.

Chart 4 is a portfolio composition map of the MM optimised asset allocations across the Michaud efficient frontier for data from Table 1 excluding gold and commodities. At very low risk, T-bills dominate. At more moderate levels of risk European government bonds are an important asset in the optimal allocation. At higher levels of risk the allocations smoothly increase for US bonds, European Large and Small Cap Equities and World ex Europe Equities. In contrast, MV optimised asset allocation would have had a minimal or non-existent role for European Bonds, US Bonds and World ex Europe Equities. The results are useful for examining the effect of gold and commodities for the base case data.[16]

Chart 5 presents the portfolio composition map of the MM optimised asset allocations including gold. Allocations to gold are substantial, running up to 10% at higher levels of risk. Importantly, the effect of introducing gold on the other assets is to reduce their allocations in roughly equal measure. In other words, the presence of gold is not a substitute for any particular asset but appears to add to portfolio optimality across the risk spectrum.

16 Note the very different qualitative results in our MM portfolio composition maps relative to those in Idzorek (2005) particularly for higher risk asset allocations. MM optimisation always computes more effectively diversified portfolios across the entire risk spectrum and avoids the unjustifiable investment limitations of one hundred percent "optimal" asset allocations in a single asset associated with MV and Feldman (2003) optimisation procedures.

Chart 4: MM strategic asset allocation composition map base case excluding gold and commodities; January 1986 to December 2010

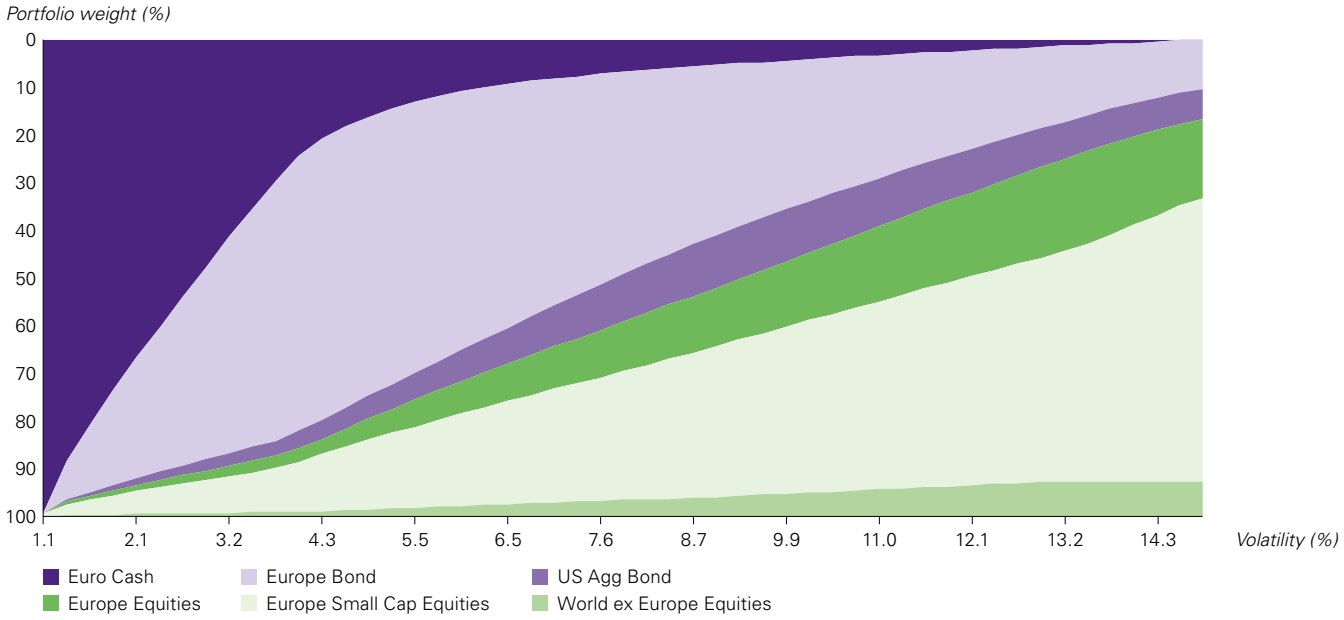


Chart 5: MM strategic asset allocation composition map base case with gold excluding commodities; January 1986 to December 2010

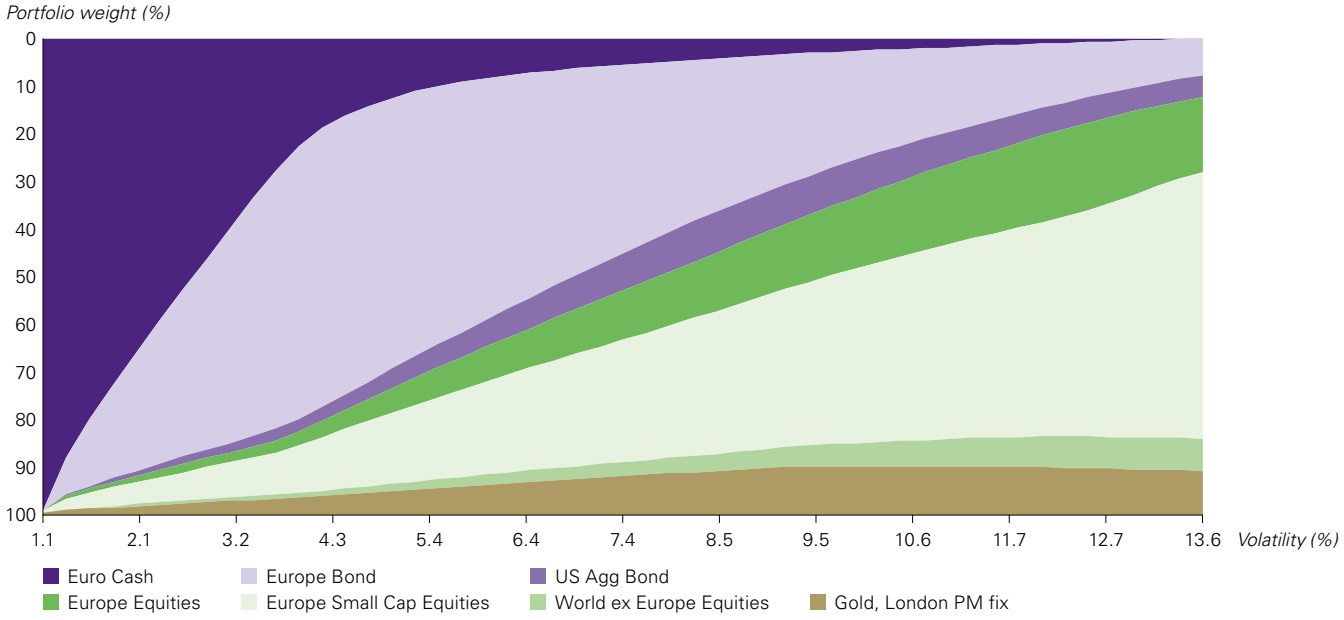


Chart 6 provides a statistical analysis of the significance of the gold. The figure displays the optimal allocation and the 25th and 75th percentile ranges of the distribution of allocations to gold across the Michaud efficient frontier. The 25% bound is greater than zero for all but the most risky portfolio. Our evidence indicates that gold is statistically significant at the 25% level for most strategic asset allocations for the last 25 years of inflation adjusted historical risk-return data.

A simple measure of the relative importance of gold in a strategic asset allocation is to compare the optimal allocation to a relatively comparable competing asset. In this comparison, gold possesses robust diversification properties relative to a similar risky asset.

Chart 7 displays the portfolio composition map of the MM optimised asset allocations including both gold and the DJ-UBS Commodity Index. Chart 8 displays the corresponding analysis of statistical significance. These results show that although the commodity futures index is a substantial component of optimal portfolios along the entire efficient frontier, including commodities with gold changes neither the significance nor the portfolio weight of gold appreciably. Gold continues to be important in defining portfolio optimality even in the presence of the commodity futures index and remains statistically significant at the 25% level for all but the highest risk portfolio.

Chart 6: Statistical significance of gold MM allocation and 25th and 75th percentiles base case, excluding commodities; January 1986 to December 2010

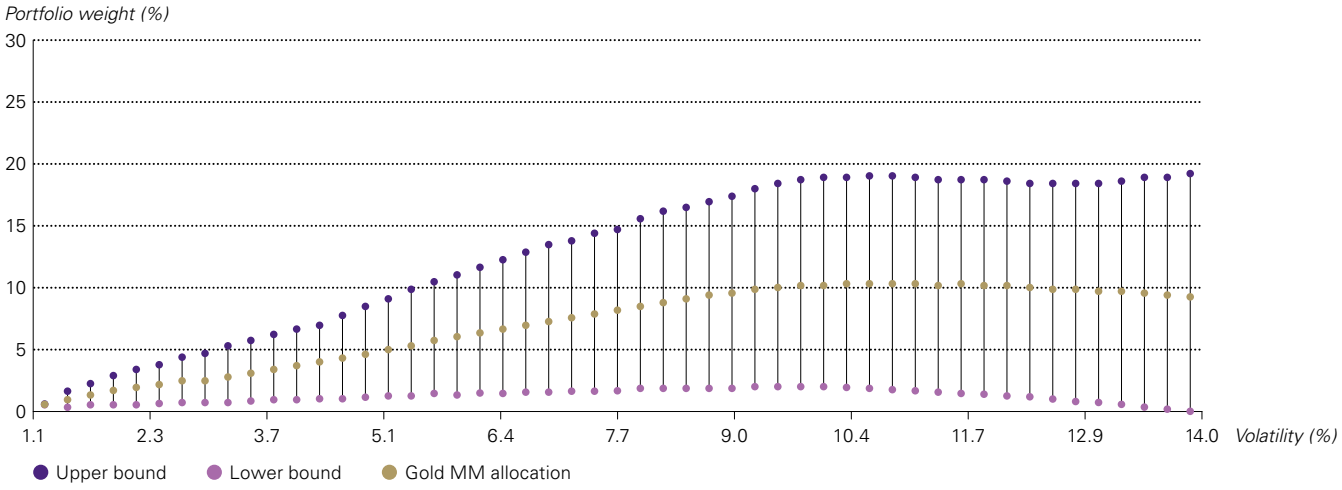


Chart 7: MM strategic asset allocation composition map base case with gold and commodities; January 1986 to December 2010

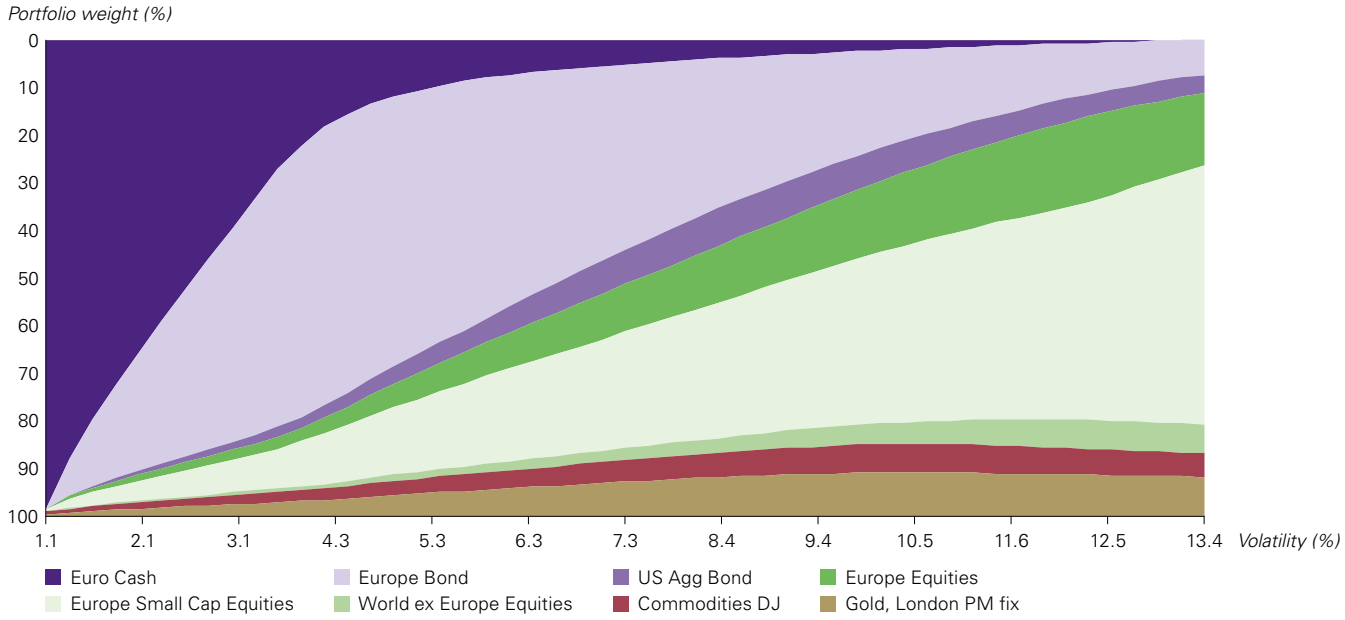
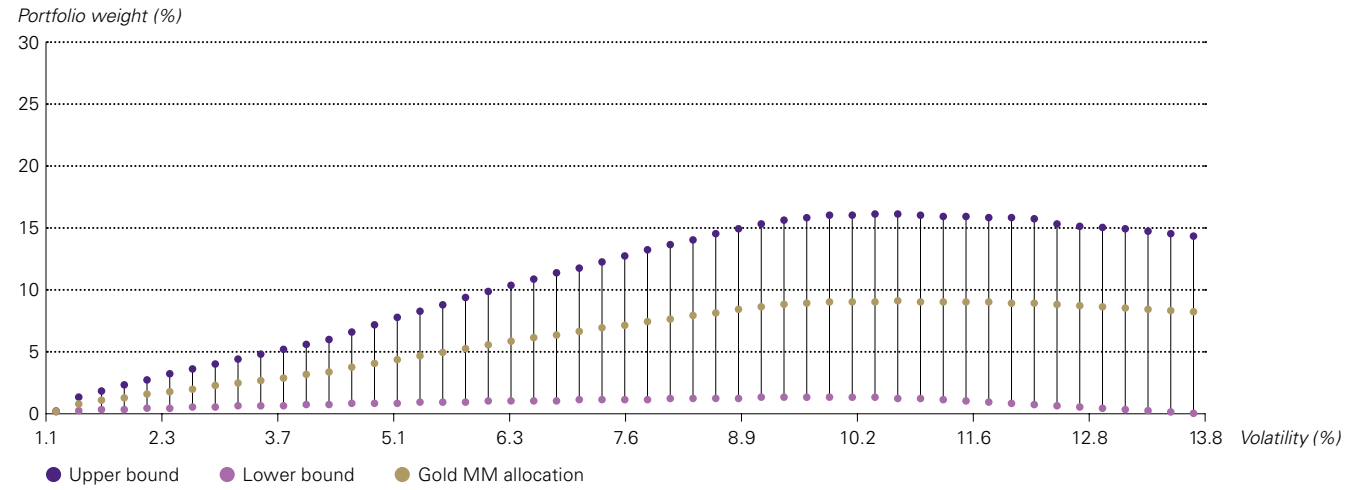


Chart 8: MM strategic base case statistical significance of gold 25th and 75th percentiles with commodities; January 1986 to December 2010



Case 2 results

We examine two cases associated with our Table 2 data set and estimated return premiums. The first represents a return premium case that excludes the effect of gold on our MM optimised asset allocations, and the second includes gold.

Chart 9 presents the portfolio composition map of the MM optimised asset allocations in Table 2 excluding gold. It is of interest to compare these results to those from historically estimated risk-returns in Chart 4. The most important differences are the diminished role of European small cap and increased role of World ex Europe.

Chart 10 presents the portfolio composition map including gold in the return premium case corresponding to Chart 9. Again, the inclusion of gold reduces the other assets' allocations roughly proportionally, indicating that gold is not substituting for any particular asset but rather providing its own diversifying benefit to the portfolio. Given a zero real return assumption, gold's smaller allocation in this case relative to Chart 5 is not surprising. Chart 11 examines the extent to which the allocation to gold is statistically significant. The exhibit indicates that gold is still statistically significant at the 25% level for all but the riskiest portfolio.

Given the conservative return estimates for gold, a strategic allocation of 2%-7% is likely to be important for many large institutional investors relative to a small set of standard indices. The optimisation results including commodities are unsurprising and not reported here. The addition of commodities to the case does not change the statistical significance of gold and has little impact on the allocation.

Chart 9: MM strategic asset allocation composition map base case with return premiums; January 1986 to December 2010

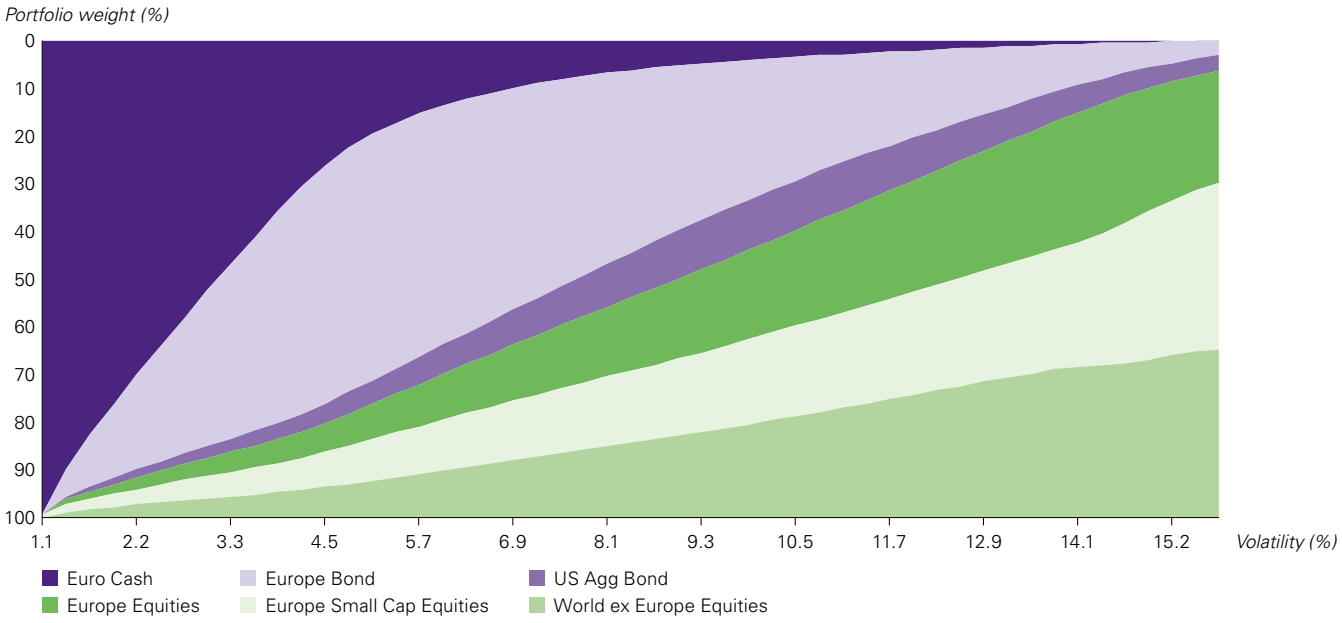


Chart 10: MM strategic asset allocation composition map base case with return premiums and gold; January 1986 to December 2010

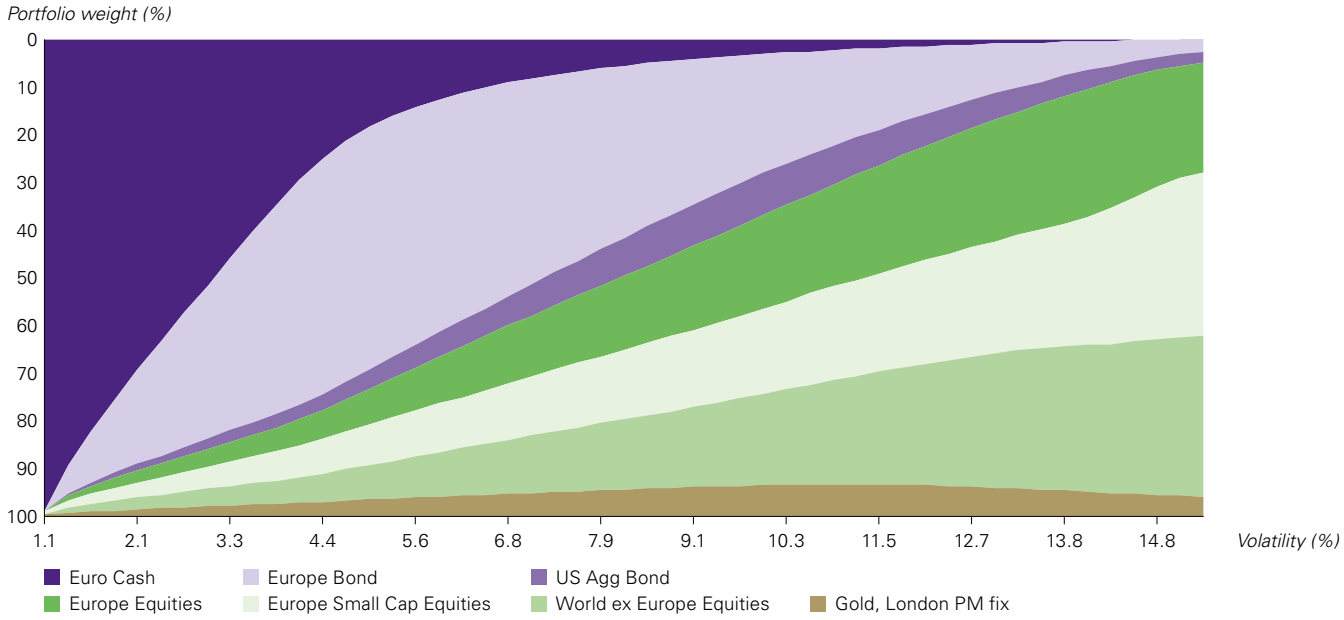
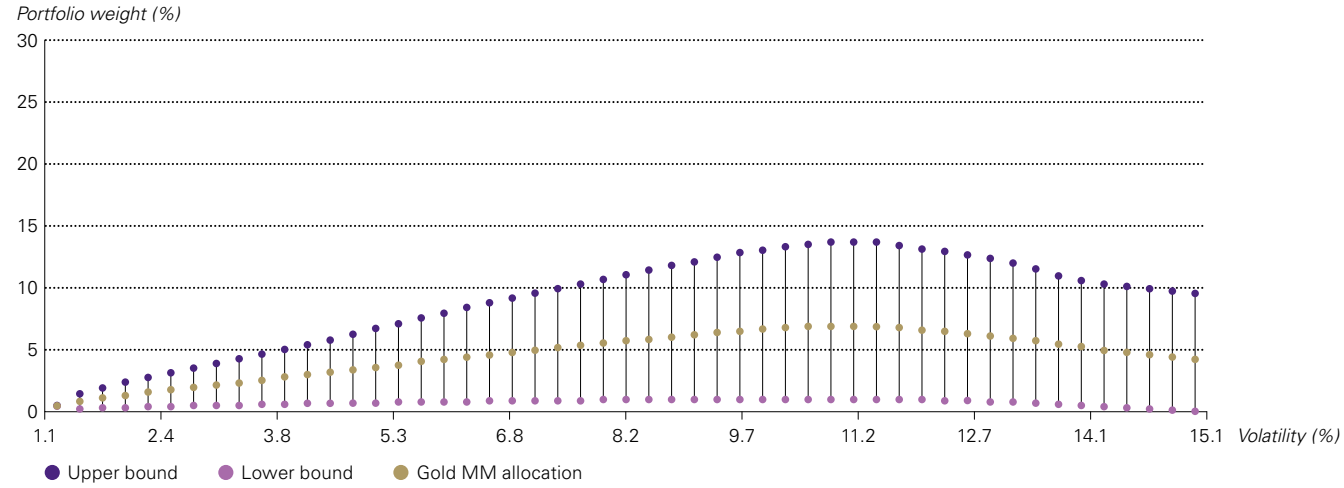


Chart 11: Statistical significance of gold MM allocation and 25th and 75th percentiles base case, return premiums excluding commodities; January 1986 to December 2010



Case 3 results

We examine the case associated with our Table 3 data set and Table A1 in Appendix I. This case represents the expanded data set with gold excluding commodities. The case including both gold and commodities has similar conclusions and allocations to the other such cases. Commodities provide insignificant benefits to the portfolio already containing gold and moreover, all of the allocations to commodities are small. We omit the results of the case including commodities for brevity.

The Chart 12 composition map and Chart 13 associated statistical significance of the allocations to gold indicate optimal allocations to gold ranging from 0.5% at low-risk levels to nearly 4% at intermediate-risk. It should not be surprising that the level and significance of an optimal allocation to gold is diminished in the context of an expanded set of investable assets relative to Charts 5 and 6. The allocations represent a conservative allocation to gold that may be investment significant for large institutional investors.

Chart 12: MM strategic asset allocation composition map expanded case with gold; January 1986 to December 2010

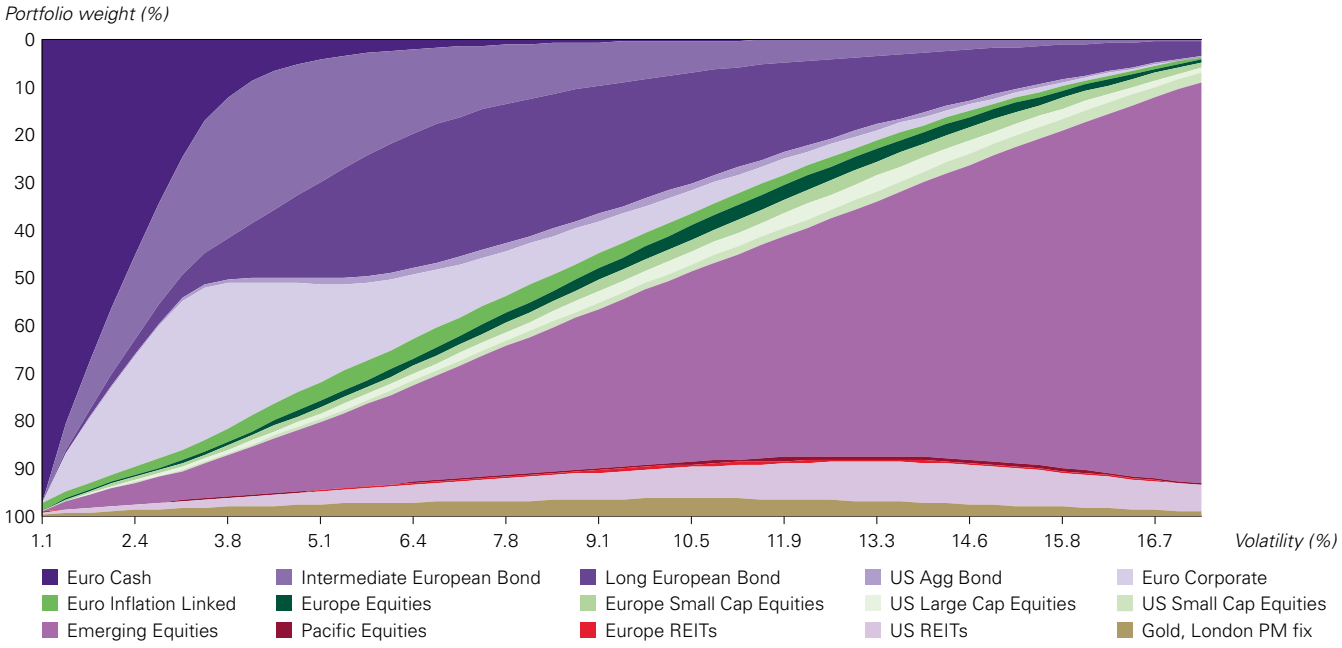
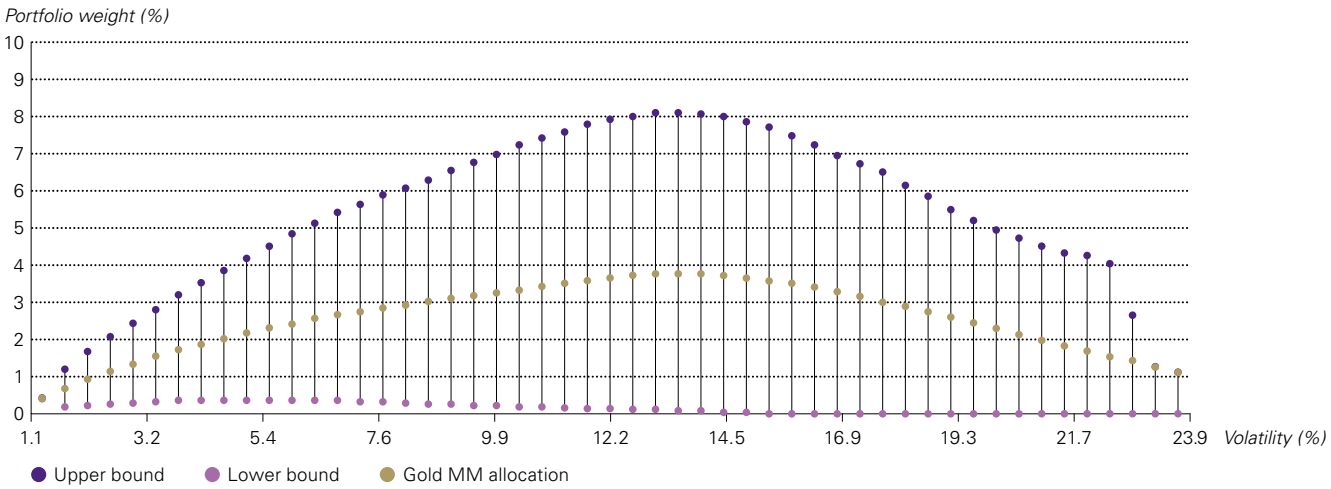


Chart 13: Statistical significance of gold MM allocation and 25th and 75th percentiles expanded case; January 1986 to December 2010



Case 4 results

We examine the case associated with our return premiums for the expanded asset class case from Tables 4 and A1 in Appendix I, including gold and excluding commodities. The case including both gold and commodities, once again, is nearly identical to the case without commodities, and provides little further insight.

The Chart 14 composition map and the Chart 15 statistical analysis chart indicates a 0.5%-4% optimal allocation to gold is statistically significant at the 25% level for the lower half of the frontier. Note that gold competes reasonably well with European equities and European real estate assets as an alternative diversifying asset at low and moderate-risk in spite of a much smaller real return assumptions.

Chart 14: MM strategic asset allocation composition map expanded case with return premiums and gold; January 1986 to December 2010

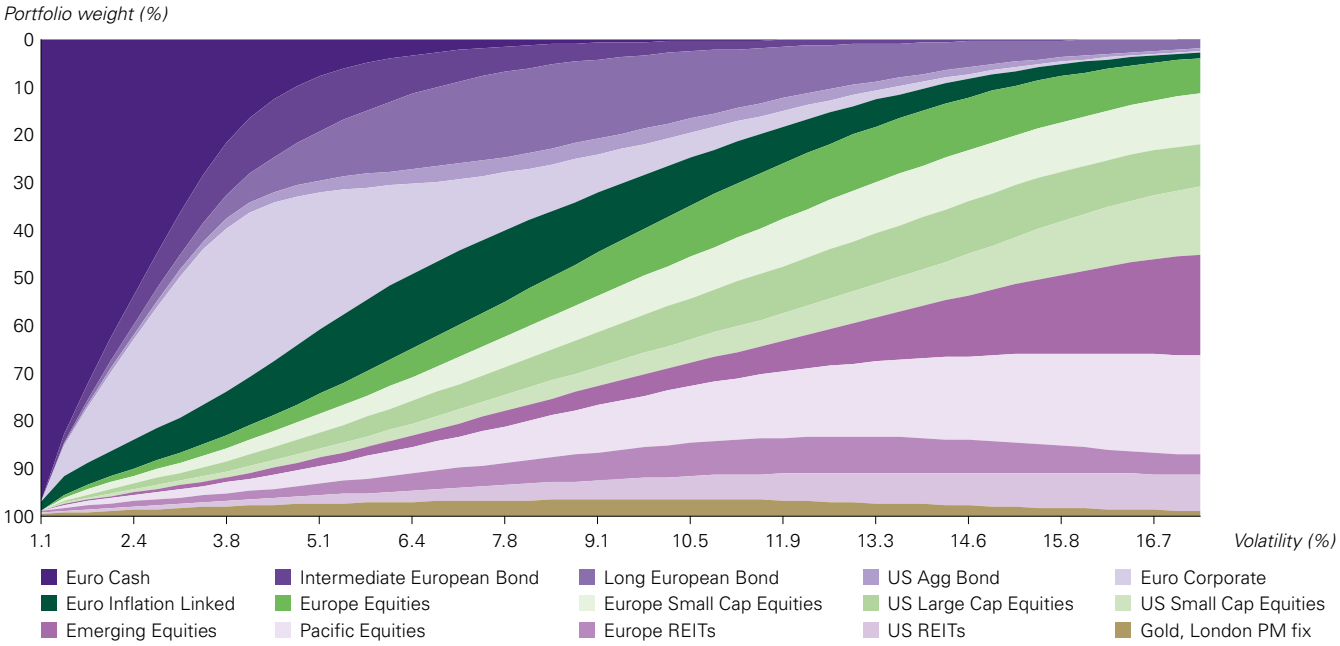
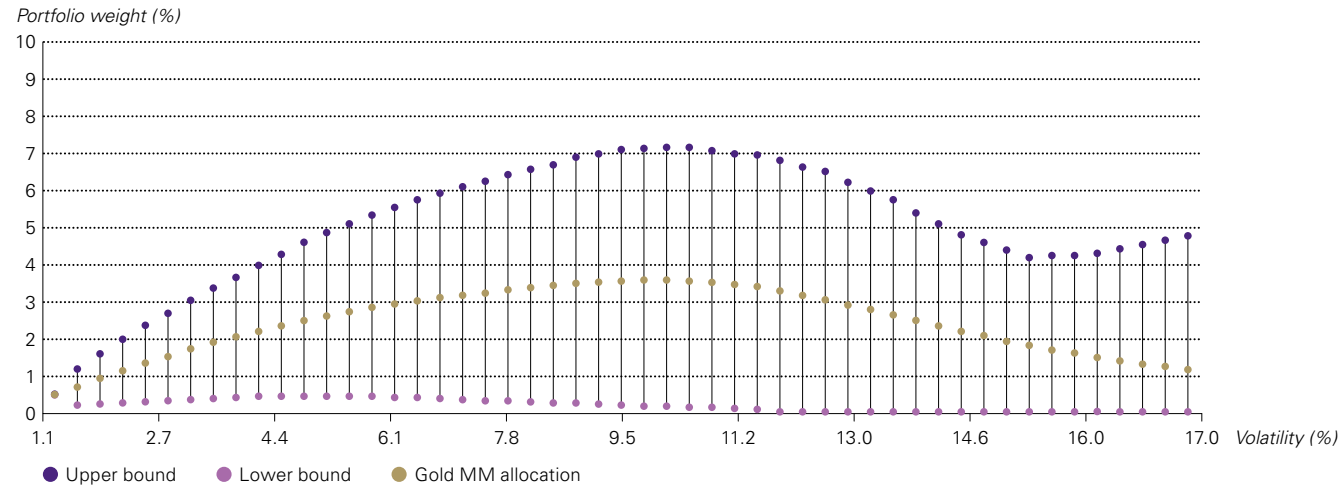


Chart 15: Statistical significance of gold MM allocation and 25th and 75th percentiles expanded case with return premiums; January 1986 to December 2010



Case 5 results

We examine the case associated with our Table 5 data set for the more recent 12-year period. This case includes gold and excludes commodities. Chart 16 displays the MM portfolio composition map and Chart 17 presents the statistical analysis of gold for the recent data. Even in this period when gold was the highest performing asset for the data set, setting the real return to zero resulted in similar allocations and statistical levels of quartile significance associated with other cases in this report.

Gold remains significant at the 25% level along the lower half of the frontier up until the higher-risk levels. The corresponding results when commodities are also included support the same conclusions as all of the earlier exhibits, namely, that commodities are not a viable substitute for a gold allocation, and have little impact on gold's role as a diversifying asset in an institutional investor portfolio. We omit these results for brevity.

Chart 16: MM strategic asset allocation composition map expanded case with gold; January 1999 to December 2010

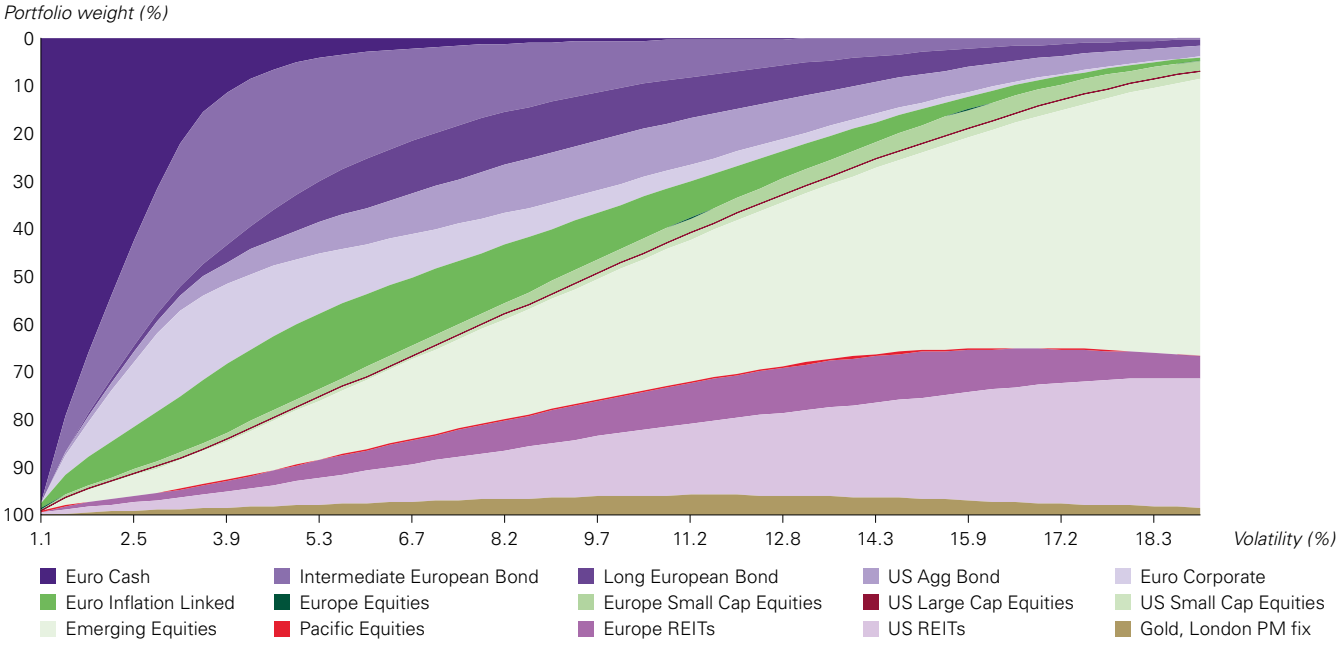
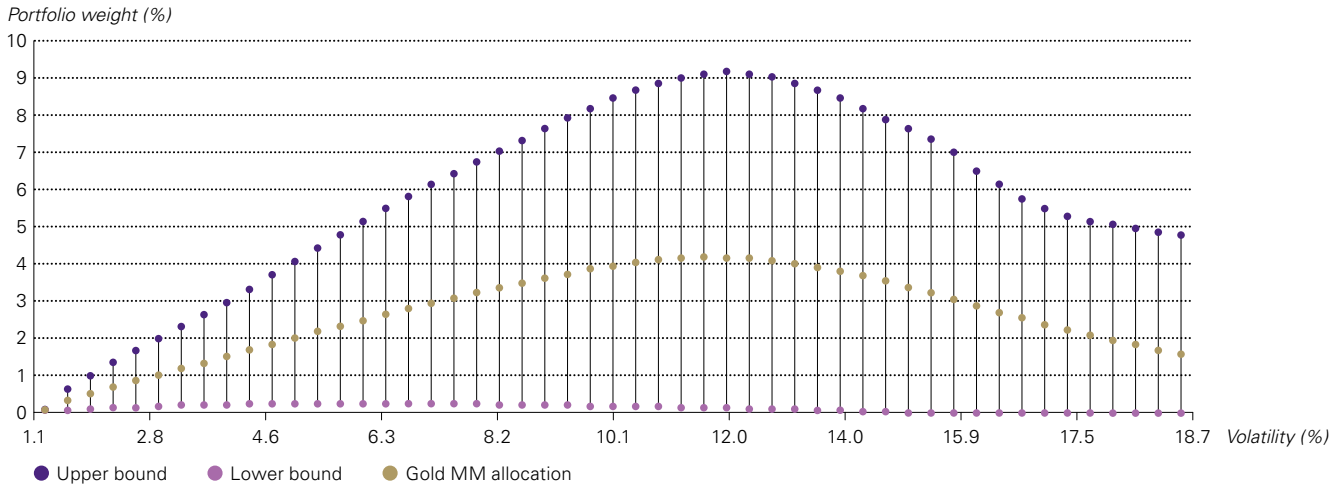


Chart 17: Statistical significance of gold MM allocation and 25th and 75th percentiles expanded case; January 1999 to December 2010



V: Summary and conclusion

Our objective was to examine the case for gold as a diversifying asset in the context of a common currency European institutional strategic asset allocation. We examine five cases of euro area based strategic asset allocations. The cases include empirical data from January 1986 through 2010 and more recent data. An eight asset case comprises basic classes likely to be part of many asset allocation optimisations and an expanded set of classes that may more relevantly reflect institutional practice. Our empirical estimates use modern methodologies for enhancing reliability. These results are supplemented with conservative return premium assumptions that include assuming that gold and commodities had zero real returns. All optimised portfolio allocations in this study were computed using Michaud optimisation, an extension of Markowitz MV optimisation that provides provably enhanced out-of-sample performance and allows the analysis of the statistical significance of an individual asset within an optimised frontier portfolio.

The results show that gold has a strategic diversifying role roughly comparable to risky assets such as small cap and emerging markets over the long-term. The results indicate nonzero mean allocations and generally nonzero lower quartile values except at some high-risk levels. Consistent with prior results in Michaud et al. (2006), gold does not appear to be a substitute for other assets but seems to add significant diversifying power due to its low or negative correlation with most other asset classes in an optimised portfolio context. These findings hold with data restricted to a more recent time period and across basic and expanded asset universes.

We find that an appropriate allocation to gold depends on risk level and competing assets in the optimisation universe. A relatively small allocation to gold appears to add useful and likely significant diversification benefits for low to moderate strategic risk levels. The results are most favourable for gold in the two relatively institutionally limited basic assets cases. In the three more institutionally relevant expanded assets cases the role of gold is diminished but not insignificant under our assumptions. Our findings indicate an allocation of 1%-3% at low to moderate risk levels may be appropriate for many strategic institutional euro area portfolios. For high-risk portfolios some limited evidence for gold is available from our results. However, a small allocation to gold may be sensible in a balanced or even riskier portfolio since it is generally advisable to include all sources of economic risk in a global strategic asset allocation. In all cases our results should be interpreted as rough guidelines and not exact recommendations.

Appendix I: Statistical procedures; mean and variance estimates

Several innovative statistical procedures have been used in this study. These techniques are proven in theory and further confirmed by practical research to provide superior results in asset allocation. The following sections provide brief overviews of these procedures. The final section of the appendix shows the mean and variance estimates for the complete set of asset classes included in all of the cases in this study.

A1 Estimation with missing data

The missing data algorithm used is a full-information maximum likelihood estimation (MLE) technique. MLE uses the complete set of observed data to holistically estimate mean and variance, taking advantage of all linear relationships among the variables to consistently estimate all of the means, standard deviations, and correlations across the entire time period spanned by the dataset. In cases with complicated patterns of missing data, the Expectation-Maximization (EM) Algorithm (Dempster, Laird, Rubin 1977) is necessary to perform this calculation, but in the simpler cases presented here, when missing data can be arranged in an order such that each successive variable's missing observations is a subset of the last, MLE can be performed non-iteratively based on a conditional factoring of the full likelihood formula.

The maximum likelihood approach to missing data is useful since it makes full use of all the information from the data, and always produces coherent estimates. Estimates on variables with missing data are updated with information from other variables to MLE the missing time periods. Covariance estimates are always positive-definite and do not cause any numerical issues in subsequent steps of the analysis. This method is the most suitable statistical practice for analysis here and will produce the best possible allocations for the data.

A2 The *t* distribution

We use the Student *t* distribution (Student, 1908) to obtain plausible mean-variance assumptions for each resampled case, based on the observed data. This probability distribution properly uses the data to take account of the unknown nature of the true mean and variance. It provides greater dispersion than the normal distribution, and thus greater coverage of possible out-of-sample scenarios. Research has shown it to produce better performance out-of-sample than the normal distribution, especially in lower-risk portfolios on the frontier.

A3 MM optimisation using the Critical Line Algorithm (CLA)

As previously indicated, the MM optimisation procedure works by averaging together many MV frontiers. Each frontier can be produced by separately optimising each desired point on the frontier with a suitable quadratic programming algorithm, or the entire frontier can be deduced using Markowitz's original Critical Line Algorithm (Markowitz 1956, 1959). Once the entire frontier is known, all frontier points are available as linear interpolations between pivot portfolios. Although the CLA itself is not new, the implementation used in this paper is relatively recent. Research has found the CLA to be faster in most cases than the separate optimisation, while providing plenty of accuracy. All optimisations shown in this work use the CLA.

A4 Subdividing efficient frontiers by arc length

When MV frontier portfolios are averaged together to create a MM frontier, points must be selected to represent each frontier in the average. The easiest computation by far is to select points with equally spaced expected returns on each frontier. Research has found, however, that dividing each frontier into segments of equal arc length, while more costly computationally, produces better out-of-sample outcomes than spacing by means (Esch 2011). For this reason arc-length spacing for all MV optimisations has been used throughout this research.

A5 Complete tables of expected returns and correlations for expanded case, including auxiliary assets for MLE estimation

Table A1

Asset name	Index	From	Mean	Std. Dev.
JPM Euro 3-Month T-bill	JPM EC: ECU/Euro 3-Month Total Return Index	Nov-86	2.6%	1.1%
JPM GBI European 5-7 Year	JPM GBI Global European 5-7 Years Total Return Index	Jan-95	4.5%	4.4%
JPM GBI European 10+ Year	JPM GBI Global European 10+ Years Total Return Index	Jan-95	5.6%	8.0%
JPM GBI European Bond	JPM GBI Global European Traded Index Total Return Index	Feb-86	4.4%	4.6%
JPM GBI Global European 1-5 Year	JPM GBI Global European 1-5 Years Total Return Index	Feb-86	3.6%	3.1%
JPM GBI Global European 5-10 Year	JPM GBI Global European 1-10 Years Total Return Index	Feb-86	4.2%	4.0%
BarCap US Agg Bond	BarCap Aggregate Bond Index Level TR	Jan-86	3.6%	10.8%
BofA Merrill Lynch US Corporate	BofA Merrill Lynch US Corporate AA-AAA TR Index	Apr-87	3.7%	11.0%
JPM Euro Corporates	JPM Corporates AAA-AA All Index Level	Jan-99	4.2%	3.8%
BarCap Euro Inflation Linkers	BarCap Euro Overall All Maturities TR	Jan-00	2.9%	7.3%
MSCI Europe	MSCI Europe	Jan-86	6.7%	16.9%
MSCI Europe Small	MSCI Europe Small	Jan-93	6.7%	18.7%
Russell 1000 US Large	Russell 1000® Index	Jan-86	7.2%	18.9%
Russell 2000 US Small	Russell 2000® Index	Jan-86	7.2%	22.5%
MSCI Emerging	MSCI Emerging Markets	Jan-88	13.8%	26.6%
MSCI Pacific	MSCI Pacific	Jan-86	3.9%	21.1%
MSCI World ex Europe	MSCI World ex Europe	Jan-86	5.0%	17.5%
European Real Estate	FTSE EPRA/NAREIT Developed Europe TR Index	Jan-90	3.4%	16.1%
US REITs	DJ US Select REIT Index	Jan-86	6.9%	20.9%
Commodities, DJ-UBS CI	DJ-UBS Commodity Index TR	Jan-91	4.2%	15.5%
Gold, London PM fix	London PM Gold Spot euro/oz	Jan-86	2.9%	15.7%
JPM 1-3 Year Germany	JPM WG: Germany 1-3 Years Total Return Index	Nov-86	2.8%	2.9%
JPM 5-7 Year Germany	JPM WG: Germany 5-7 Years Total Return Index	Jan-86	4.4%	4.6%
JPM 7-10 Year Germany	JPM WG: Germany 7-10 Years Total Return Index	Jan-86	4.3%	5.5%

Table A2

	Correlations											
	3-Month T-bill	JPM Eur 5-7 yr	JPM Eur 10+ yr	JPM Euro Bond	JPM Eur 1-5 yr	JPM Eur 1-10 yr	US Agg	US Corp	Euro Corp	Euro Inflation	Europe Equities	Europe Small Equities
3-Month T-bill	1.00											
JPM Eur 5-7 Year	0.42	1.00										
JPM Eur 10+ Year	0.37	0.79	1.00									
JPM Euro Bond	0.46	0.88	0.88	1.00								
JPM Eur 1-5 Year	0.55	0.83	0.75	0.86	1.00							
JPM Eur 1-10 Year	0.48	0.88	0.84	0.91	0.90	1.00						
US Agg	0.20	0.30	0.36	0.34	0.35	0.34	1.00					
US Corp	0.22	0.33	0.40	0.38	0.37	0.37	0.93	1.00				
Euro Corp	0.43	0.81	0.64	0.74	0.70	0.74	0.16	0.25	1.00			
Euro Inflation	0.04	0.49	0.27	0.32	0.24	0.31	-0.12	-0.08	0.56	1.00		
Europe Equities	0.04	0.10	0.22	0.16	0.15	0.16	0.23	0.26	0.11	-0.01	1.00	
Europe Small Equities	-0.01	0.02	0.15	0.09	0.08	0.08	0.18	0.21	0.06	-0.04	0.86	1.00
Russell 1000 Equities	0.08	0.08	0.19	0.14	0.16	0.14	0.55	0.56	0.04	-0.14	0.77	0.66
Russell 2000 Equities	0.03	0.03	0.11	0.08	0.10	0.08	0.44	0.45	0.00	-0.12	0.72	0.69
Emerging Equities	0.04	0.02	0.10	0.08	0.10	0.07	0.31	0.32	0.01	-0.14	0.70	0.70
Pacific Equities	-0.01	0.12	0.19	0.17	0.17	0.16	0.23	0.25	0.08	0.08	0.59	0.57
World ex Europe Equities	0.05	0.11	0.22	0.17	0.18	0.17	0.45	0.46	0.06	-0.05	0.80	0.70
Eur Real Estate	-0.03	0.18	0.25	0.22	0.18	0.21	0.14	0.17	0.18	0.13	0.65	0.70
US REITs	-0.06	0.05	0.14	0.10	0.08	0.09	0.40	0.42	0.03	0.02	0.52	0.54
DJ-UBS CI	-0.06	-0.03	0.05	0.02	0.05	0.02	0.38	0.39	-0.07	-0.11	0.31	0.35
Gold, London PM fix	0.00	0.04	0.05	0.06	0.10	0.06	0.37	0.36	-0.02	-0.04	0.01	0.04
JPM 1-3 Year Germany	0.31	0.55	0.19	0.35	0.40	0.38	0.02	0.05	0.63	0.69	-0.13	-0.19
JPM 5-7 Year Germany	0.30	0.78	0.53	0.63	0.57	0.63	0.14	0.17	0.78	0.67	-0.10	-0.17
JPM 7-10 Year Germany	0.28	0.78	0.62	0.67	0.56	0.66	0.14	0.18	0.77	0.67	-0.09	-0.14

	Russell 1000 Equities	Russell 2000 Equities	Emerging Equities	Pacific Equities	World ex Europe Equities	Eur Real Estate	US REITs	DJ-UBS CI	Gold, London PM	JPM 1-3 yr Germ	JPM 5-7 yr Germ	JPM 7-10 yr Germ
3-Month T-bill												
JPM Eur 5-7 Year												
JPM Eur 10+ Year												
JPM Euro Bond												
JPM Eur 1-5 Year												
JPM Eur 1-10 Year												
US Agg												
US Corp												
Euro Corp												
Euro Inflation												
Europe Equities												
Europe Small Equities												
Russell 1000 Equities	1.00											
Russell 2000 Equities	0.86	1.00										
Emerging Equities	0.72	0.72	1.00									
Pacific Equities	0.53	0.50	0.60	1.00								
World ex Europe Equities	0.89	0.79	0.75	0.80	1.00							
Eur Real Estate	0.48	0.48	0.47	0.50	0.56	1.00						
US REITs	0.64	0.70	0.51	0.36	0.58	0.57	1.00					
DJ-UBS CI	0.43	0.41	0.45	0.38	0.45	0.27	0.34	1.00				
Gold, London PM fix	0.14	0.17	0.22	0.20	0.18	0.03	0.16	0.39	1.00			
JPM 1-3 Year Germany	-0.15	-0.14	-0.17	-0.08	-0.14	-0.07	-0.12	-0.17	0.00	1.00		
JPM 5-7 Year Germany	-0.11	-0.14	-0.16	-0.04	-0.10	-0.01	-0.08	-0.17	0.00	0.78	1.00	
JPM 7-10 Year Germany	-0.11	-0.15	-0.18	-0.05	-0.10	0.01	-0.08	-0.18	-0.02	0.69	0.88	1.00

Appendix II: List of charts and tables

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World Gold Council
10 Old Bailey, London EC4M 7NG
United Kingdom

T +44 20 7826 4700
F +44 20 7826 4799
W www.gold.org

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